

# D-BIonic Screening of Scalar Fields

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We study a new screening mechanism which is present in Dirac-Born-Infeld (DBI)-like theories. A scalar field with a DBI-like Lagrangian is minimally coupled to matter. In the vicinity of sufficiently dense sources, non-linearities in the scalar dominate and result in an approximately constant acceleration on a test particle, thereby suppressing the scalar force relative to gravity. Unlike generic  $P(X)$  theories, screening happens within the regime of validity of the effective field theory, thanks to the DBI symmetry. This symmetry also allows the removal of a constant field gradient, like in galileons. Not surprisingly, perturbations around the spherically-symmetry background propagate superluminally, but we argue for a chronology protection analogous to galileons. We derive constraints on the theory parameters from tests of gravity and discuss various extensions.

The discovery of cosmic acceleration has generated much activity to explore new physics associated with dark energy. The central motivation for “Beyond- $\Lambda$ CDM” physics is of course the smallness of the cosmological constant. But another, more pragmatic motivation is the opportunity offered by upcoming surveys of the large scale structure, such as the Euclid mission and the Large Synoptic Survey Telescope, which will subject the  $\Lambda$ CDM model to its most stringent tests to date.

The most exciting and phenomenologically-rich possibility is if the new sector associated with dark energy (usually in the form of light scalar fields) couple to matter. For consistency with laboratory and solar system tests of gravity, however, any such scalar field must become invisible in the local environment. This is achieved through *screening mechanisms* [1, 2]. In regions of high density/curvature, the scalar  $\phi$  develops non-linearities, which in turn decouple it from matter.

Screening mechanisms can be classified according to the nature of their non-linearities [3]:

- *$\phi$  screening.* In this case, self-interactions are governed by a potential  $V(\phi)$ . Whether or not the scalar develops non-linearities depends on the local value of  $\phi$ . This includes chameleon [5–7], symmetron [8–10] and varying-dilaton theories [11]. Quite generally, the cosmological effects of such scalars are restricted to non-linear ( $\lesssim$  Mpc) scales [12], and the lack of any symmetry makes them susceptible to radiative corrections [13]. There are also issues with initial conditions [14, 15]. Since  $\phi$  is canonical, on the other hand, this class of theories does not suffer from superluminality issues and in principle admits a standard Wilsonian UV completion [16].
- *$\partial\phi$  screening.* These theories are of the  $P(X, \phi)$  type, where  $X = -(\partial\phi)^2/2$ . The threshold for screening is set by the local gradient  $\partial\phi$ . This includes  $k$ -mouflage [17] and generic  $P(X)$  theories [18].
- *$\partial^2\phi$  screening.* Non-linearities are triggered in this case by the local acceleration  $\partial^2\phi$  exceeding some threshold. This mechanism originated in massive gravity [19] to resolve the van Dam–Veltman–Zakharov (vDVZ) discontinuity [20]. This class includes galileons [21–23], which are invariant under a linear-gradient shift  $\delta\phi = c + b_\mu x^\mu$ ,

as well as massive/resonance gravitons, *e.g.*, [24, 25].

Phenomenologically,  $P(X)$  theories are similar to galileons. A key difference, however, is that the screened regime for  $P(X)$  generally lies outside the regime of validity of the effective field theory (EFT). Consider  $P(X) = X + \alpha X^2$ , for instance. Screening requires  $X^2$  to dominate over the kinetic term, at which point all sorts of higher-dimensional operators (*e.g.*,  $X^n$ ) should become important.

An important exception, which is the focus of this Letter, is based on the Dirac-Born-Infeld (DBI) action

$$\mathcal{L} = \Lambda^4 \sqrt{1 - \Lambda^{-4}(\partial\phi)^2}. \quad (1)$$

Compared to the standard DBI form, we have flipped both the overall sign of the action, together with the sign of the  $(\partial\phi)^2$  coefficient. This is necessary to achieve screening, as we will see. We will refer to the screening mechanism in this case as *D-BIonic screening*. This was first briefly considered in [26]. Expanding around  $\phi = 0$ ,  $\mathcal{L} = \Lambda^4 - (\partial\phi)^2/2 - (\partial\phi)^4/8\Lambda^4 + \dots$ , we see that the kinetic term has the correct sign, while the quartic term has the wrong-sign for S-matrix analyticity [27].

The action (1) can be interpreted as the area of a (negative-tension) 3-brane moving in a 5d Minkowski space with 2 time dimensions. The brane position is defined by  $\phi(x)$ , and the induced metric on the brane is

$$\tilde{g}_{\mu\nu} = \eta_{\mu\nu} - \Lambda^{-4} \partial_\mu \phi \partial_\nu \phi. \quad (2)$$

In terms of this metric,  $\mathcal{L} = \Lambda^4 \sqrt{-\tilde{g}}$ . The theory is therefore invariant under 5d Lorentz boosts ( $\gamma \equiv 1/\sqrt{1-v^2}$ ):

$$\begin{aligned} \tilde{\phi}(\tilde{x}) &= \gamma(\phi(x) + \Lambda^2 v_\mu x^\mu); \\ \tilde{x}^\mu &= x^\mu + \frac{\gamma-1}{v^2} v^\mu v_\nu x^\nu + \gamma v^\mu \frac{\phi(x)}{\Lambda^2}. \end{aligned} \quad (3)$$

This symmetry distinguishes DBI from a generic  $P(X)$  theory in several important ways. Much like in DBI inflation [28],  $\phi$  can acquire large gradients  $|\vec{\nabla}\phi| \gg \Lambda^2$  while remaining within the regime of validity of the EFT, as long as the proper acceleration remains small:

$|a_{\text{brane}}| = \gamma|\vec{\nabla}^2\phi| \ll \Lambda^3$ . Second, a constant gradient profile can be removed by a “boost” and is therefore unobservable, similar to galileons. Indeed, the DBI mechanism is intermediate between  $P(X)$  and the galileon.

For simplicity, we couple  $\phi$  conformally to matter:

$$\mathcal{L}_{\text{coupling}} = \frac{g\phi}{M_{\text{Pl}}} T_{\mu}^{\mu}, \quad (4)$$

where  $g$  is dimensionless. This operator breaks the symmetry (3), but very mildly since  $\Lambda \ll M_{\text{Pl}}$ . In the concluding remarks, we will discuss more general matter couplings which could be relevant for the cosmology. The equation of motion for  $\phi$  is

$$\partial_{\mu} \left( \frac{\partial^{\mu}\phi}{\sqrt{1 - \Lambda^{-4}(\partial\phi)^2}} \right) = -\frac{g}{M_{\text{Pl}}} T_{\mu}^{\mu}. \quad (5)$$

*Static profile:* Around a static point source,  $T_{\mu}^{\mu} = -\rho = -M\delta^{(3)}(\vec{x})$ , the scalar field profile can be assumed static and spherically symmetric. The equation of motion can then be integrated once to give

$$\frac{E}{\sqrt{1 - E^2}} = \frac{gM}{4\pi M_{\text{Pl}}\Lambda^2 r^2}, \quad (6)$$

where  $E \equiv \phi'/\Lambda^2$ , and  $' \equiv d/dr$ . This implies [26]

$$E = \frac{1}{\sqrt{1 + (r/r_*)^4}}, \quad (7)$$

where we have introduced the “Vainshtein” scale

$$r_* \equiv \frac{1}{\Lambda} \left( \frac{gM}{4\pi M_{\text{Pl}}} \right)^{1/2}, \quad (8)$$

analogously to galileon/massive gravity. Note that  $r_* \sim M^{1/2}$ , instead of the usual  $\sim M^{1/3}$  for massive gravity.

The scalar force on a test mass is  $\vec{F}_{\phi} = -gm\vec{\nabla}\phi/M_{\text{Pl}}$ . At large distances from the source,  $r \gg r_*$ ,

$$\frac{F_{\phi}}{F_{\text{N}}} = 2g^2; \quad (r \gg r_*). \quad (9)$$

This is exactly the form we would expect for the force mediated by a massless scalar with coupling strength  $g/M_{\text{Pl}}$ . This can only be made weaker than the gravitational force by tuning  $g$  to be small. Close to the source,  $r \ll r_*$ , however,  $F_{\phi}$  saturates to a constant, such that

$$\frac{F_{\phi}}{F_{\text{N}}} = 2g^2 \left( \frac{r}{r_*} \right)^2; \quad (r \ll r_*). \quad (10)$$

Thus, within  $r_*$  the scalar force is suppressed compared to gravity, much like in the Vainshtein mechanism.

As mentioned earlier, even though  $\phi' \sim \Lambda^2$  close to the source, the EFT description is still valid provided that the proper “acceleration” is small:

$$|a_{\text{brane}}| = \gamma|E'| = \frac{1}{r_*} \frac{2r/r_*}{1 + (r/r_*)^4} \lesssim \Lambda. \quad (11)$$

This is satisfied for all  $r$  provided that  $r_*\Lambda \sim M/M_{\text{Pl}} \gtrsim 1$ , which is the case for any macroscopic source of interest.

*Blanket screening:* Because of non-linearities, the solutions due to different objects cannot be linearly superimposed within  $r_*$ . Instead, in the presence of a background profile for a massive object, the scalar field around a second, less massive object is renormalized.

Consider two spherical sources with masses  $M_1 \gg M_2$ , separated by a distance  $R$ . If  $R \gg r_*^{(1)}$ , then we remain in the linear regime and the solutions can be superimposed. More interesting is the regime  $R \ll r_*^{(1)}$ . The profile due to  $M_1$  can be treated as a background, with

$$\bar{\phi} \simeq \Lambda^2 r; \quad \bar{\gamma} = \frac{1}{\sqrt{1 - (\partial\bar{\phi})^2/\Lambda^4}} \simeq \left( \frac{r_*^{(1)}}{r} \right)^2. \quad (12)$$

The perturbation  $\varphi = \phi - \bar{\phi}$  due to  $M_2$ , treated as a point source, is governed by the quadratic action

$$\mathcal{L}_{\varphi} = -\frac{1}{2}G^{\mu\nu}\partial_{\mu}\varphi\partial_{\nu}\varphi + \frac{g}{M_{\text{Pl}}}\varphi M_2\delta^3(\vec{x} - \vec{R}), \quad (13)$$

where  $G^{\mu\nu} = \bar{\gamma}g^{\mu\nu} + \frac{\bar{\gamma}^3}{\Lambda^4}\partial^{\mu}\bar{\phi}\partial^{\nu}\bar{\phi}$ . In the static limit the equation for  $\varphi$  is

$$\frac{x_i x_j}{r^2} \partial^i \partial^j \varphi - 4 \frac{x_i}{r^2} \partial^i \varphi = -\frac{gM_2}{M_{\text{Pl}}\bar{\gamma}^3(R)} \delta^3(\vec{x} - \vec{R}), \quad (14)$$

where  $i, j = 1, 2, 3$  indicate Cartesian coordinates. Thus the presence of non-linear background due to a massive source suppresses the matter coupling

$$g_{\text{eff}} = \frac{g}{\bar{\gamma}^3(R)} \ll g. \quad (15)$$

Indeed, the solution to (14), obtained by changing to coordinates centred around  $M_2$  and integrating, is

$$\varphi(\tilde{r}) = \frac{gM_2}{4\pi M_{\text{Pl}}\bar{\gamma}^3(R)\tilde{r}}, \quad (16)$$

where  $\tilde{r}$  is the radial distance from  $M_2$ . A similar suppression of the renormalized coupling also occurs for galileons [22]. We refer to this as *blanket screening*.

The  $r_*$ -scale for  $M_2$  also gets renormalized by the presence of  $M_1$ . Looking at a typical interaction term for  $\varphi$ , it is straightforward to see that

$$r_*^{(2),\text{eff}} = \frac{1}{\Lambda} \left( \frac{g_{\text{eff}} M_2}{4\pi M_{\text{Pl}}} \right)^{1/2} = \frac{r_*^{(2)}}{\bar{\gamma}^{3/2}(R)} \ll r_*^{(2)}. \quad (17)$$

As with galileons, the  $r_*$ -radius is renormalized to a smaller value, meaning that non-linearities are pushed to much shorter scales.

*Solar System Constraints:* We now turn to constraints on the theory from solar system tests of gravity. Since we assume  $g \sim \mathcal{O}(1)$ , the only free parameter is the strong coupling scale  $\Lambda$ . In order to be consistent with planetary motions within the solar system, we impose that the

screening radius of the Sun, obtained by substituting the mass of Sun in (8),

$$r_*^\odot \sim 10^{14} \left( \frac{\text{eV}}{\Lambda} \right) \text{ cm}, \quad (18)$$

is larger than the size of the solar system ( $\sim 10^{15}$  cm). This translates to  $\Lambda \lesssim 0.1$  eV.

In addition, within the solar system the planets experience a constant scalar acceleration  $|a_\phi| \sim g\Lambda^2/M_{\text{Pl}}$ . We must ensure that this constant acceleration component does not significantly perturb planetary orbits. NASA's Viking mission provided radio-ranging measurements to an accuracy of about 12 m [29]. If a planet experiences, in addition to the gravitational force, a small acceleration  $\Delta a$ , its orbital radius will be perturbed by [30]

$$\frac{\Delta r}{r} \simeq -\frac{\Delta a}{a_N}, \quad (19)$$

where  $a_N$  is the Newtonian acceleration at a distance  $r$  from the Sun. The data from the Viking mission determines the difference between the orbital radii of the Earth and Mars to an accuracy of  $\sim 100$  m, and the sum of these radii to an accuracy of  $\sim 150$  m [29]. This constrains the allowed size of an additional constant acceleration in the solar system  $\Delta a \lesssim 10^{-10} \text{ cm s}^{-2}$ . For the D-Bionic component,  $|a_\phi| \sim g\Lambda^2/M_{\text{Pl}}$ , with  $g \sim \mathcal{O}(1)$ , this translates to the bound

$$\Lambda \lesssim 0.1 \text{ meV}. \quad (20)$$

Remarkably, this is of order the dark energy scale.

For these small values of  $\Lambda$ , blanket screening from the Sun makes the scalar field very weakly coupled everywhere in the solar system. For concreteness, let us take largest value allowed by (20), *i.e.*,  $\Lambda = 0.1$  meV, in which case  $r_*^\odot \simeq 10^{18}$  cm. At the location of the Earth, for instance, the  $\gamma$ -factor due to the Sun is

$$\gamma_\odot \simeq \left( \frac{r_*^\odot}{\text{AU}} \right)^2 \simeq 4 \times 10^9, \quad (21)$$

where  $\text{AU} = 1.5 \times 10^{13}$  cm. As a result, the effective matter coupling (15) is tiny,  $g_{\text{eff}} \sim 10^{-29}g$ , which renders any test of gravity involving the Earth, such as Lunar Laser Ranging (LLR), completely trivial. This is very different from (the simplest) galileons, where the Earth-Moon system can be treated in isolation, LLR yields the strongest constraint [31–33].

The constraint derived above may in fact be too conservative, once we take into account the possibility of blanket screening from the Milky Way. This is trickier to estimate, since of course the Sun lies within the galaxy. As a crude estimate of the effective  $r_*^{\text{MW}}$  at the location of the Sun, let us consider the mass enclosed within the Sun's orbit around the galactic center,  $M_{\text{enc.}} \simeq 7 \times 10^{10} M_\odot$ . Taking the fiducial value  $\Lambda = 0.1$  meV, we obtain  $r_*^{\text{MW}} \simeq 100$  kpc. Meanwhile,

our distance from the galactic center is  $\sim 8$  kpc. The  $\gamma$ -factor due to the galaxy is  $\gamma_{\text{MW}} \simeq (r_*^{\text{MW}}/8 \text{ kpc})^2 \sim 10^2$ . Thus blanket screening due to the Milky Way may be important for solar system considerations, which would weaken the bound (20). A careful analysis would require an accurate derivation of the field profile within the Milky Way. We leave this to future work, and for the moment treat (20) as a conservative limit.

*Superluminality and Chronology Protection:* Analogously to galileons, perturbations on certain backgrounds can propagate superluminally. This is usually interpreted as a sign that the UV completion is non-standard [27]. However, recent developments in galileon dualities [34–38] shows that the issue of superluminality is at the very least more subtle than anticipated. We will take a conservative point of view and, for consistency, argue for chronology protection: closed time-like curves (CTCs) cannot form within the regime of validity of the EFT.

Consider, for instance, the effective metric (13) (in spherical coordinates) for perturbations around a spherically-symmetric profile sourced by a mass  $M$ :

$$G_{\mu\nu} = \bar{\gamma}^{-1} \text{diag} \left( -1, \frac{1}{1 + \bar{\gamma}^2}, r^2, r^2 \sin^2 \phi \right). \quad (22)$$

Suppressing the angular directions, the null cones for radial propagation are defined by

$$v_\pm = \left( \pm \sqrt{1 + \bar{\gamma}^2} \right). \quad (23)$$

The null cone gets wider as  $\bar{\gamma}$  increases, allowing for superluminal propagation radially. Note, however, that the null-cone never tips over to allow propagation to negative coordinate times. There always exists a set of surfaces which are space-like with respect to both  $G_{\mu\nu}$  and  $\eta_{\mu\nu}$ , and these can be chosen as Cauchy surfaces on which to specify initial conditions. Therefore causal propagation is always possible, as in k-essence theories [39].

The reader may be concerned that Lorentz boosting the system would allow travel to negative times, and this is indeed the case. Under the Lorentz transformation  $t = \Gamma(t' + vr')$ ,  $r = \Gamma(r' + vt')$ , with  $\Gamma \equiv 1/\sqrt{1 - v^2}$  (to avoid confusion with  $\gamma$ ), the  $v_\pm$  null vectors become

$$v_\pm = \left( \gamma^2 v \pm \Gamma^{-2} \sqrt{1 + \gamma^2}, -\Gamma^{-2} - \gamma^2 \right), \quad (24)$$

where  $\gamma$  is now a function of both the Lorentz transformed coordinates  $\gamma \equiv \gamma(r' + vt')$ . The null cones tip over to include negative coordinate times when

$$\frac{\gamma^2}{\sqrt{1 + \gamma^2}} > \frac{1}{v\Gamma^2}. \quad (25)$$

For large  $\gamma$ , this becomes simply  $\gamma > 1/v\Gamma^2$ . It seems naively possible for a scalar fluctuation to propagate causally to some point  $t' < 0$ , interact with a Standard Model particle moving on geodesics of  $\eta_{\mu\nu}$ , and then

propagate forward in time, thereby creating a CTC. However it is straightforward to check that the Hamiltonian for scalar fluctuations diverges as  $\gamma \rightarrow 1/v\Gamma^2$ , hence it is not possible for the system to evolve from a situation in which no CTCs exist to one in which such curves are formed, while remaining within the regime of validity of the EFT. This is an extension of Hawking's Chronology Protection Conjecture [40] to scalar field theories with non-canonical kinetic terms, first discussed in the context of Galileon scalar field theories in [41].

*Conclusions:* In this paper we have explored a novel screening mechanism based on the DBI action, focusing primarily on constraints from local tests of gravity. Interestingly, the scale  $\Lambda$  is constrained to be of order the dark energy scale, which is promising for cosmology. A detailed analysis of the cosmological evolution is cur-

rently underway and will appear elsewhere [42]. Various extensions of the theory suggest themselves. The DBI Lagrangian (1) is only one of five DBI galileon terms, and it would be interesting to generalize the mechanism to include such terms as well. The matter coupling (4) can also be generalized. A more natural choice is to couple matter fields to a generalization of the brane metric (2),  $\tilde{g}_{\mu\nu} = e^{2g\phi/M_{\text{Pl}}} (\eta_{\mu\nu} - \Lambda^{-4} \partial_\mu \phi \partial_\nu \phi)$ . This will have no impact on the static profiles studied here, but can play an important role for the cosmological evolution.

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